Measuring Quantum Processes

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A classic analysis technique in science is to treat some unknown system or process as a "black box," and measure the properties of the system by measuring changes to a known set of inputs. In simple physical systems with few degrees of freedom, this is a straightforward matter of exhausting all possible inputs; in more complicated classical systems with many degrees of freedom, ensemble or averaging techniques may need to be used. Quantum processes are inherently different in that even the simplest processes–e.g., one that acts on a two-level quantum system, or *qubit*, an infinite number of input states are possible, and simple exhaustion of inputs and measurement of each output cannot be achieved.

For ideal quantum systems a mathematical technique, known as *quantum process tomography*, for extracting process information with a finite number of measurements have been devised. A fundamental limitation of this technique is that it only works on ideal systems. When applied to real experimental systems with associated measurement uncertainties, they can and do yield unphysical processes. This means they are useless for the purposes of measuring and evaluating performance of real quantum processes, for example, gates in quantum computation. Recently, in collaboration with the University of Queensland in Brisbane, Australia, Los Alamos National Laboratory has pioneered the notion of *physical* quantum process tomography, which yields strictly physical processes via a maximum likelihood technique, and demonstrated it experimentally using a nontrivial two-qubit process, namely an all-optical quantum CNOT gate [1].

Mathematically, a quantum process (also known as a completely positive trace-preserving map) transforms the density operator of a system according to the rule

$$\rho' = \sum_{\mu,\nu} \chi_{\mu\nu} \Gamma_\mu \rho \Gamma_\nu \ ,$$

where $\{\Gamma_\nu\}$ is a complete set of trace-orthogonal operators spanning the space of operators for the system (for a pair of qubits, these can be tensor products of two Pauli matrices), and $\chi_{\mu\nu}$ is the so-called error-correlation matrix, which uniquely specifies each process. The essential principle of quantum process tomography is shown in Fig. 1. One of a set of states, known as a *quorum*, is input into the black box. For a pair of qubits, the quorum set has at least 16 elements (i.e., the square of the dimensionality of the Hilbert space of the system). The output of the black box is then projected onto one of the states of a separate quorum. By repeating this experiment multiple times, the probability

$$p_{ab} = \sum_{\mu\nu} \chi_{\mu\nu} \langle \phi_b | \Gamma_\mu | \psi_a \rangle \langle \psi_a | \Gamma_\nu | \psi_b \rangle$$

may be deduced; repeating this process for all 16 x 16 combinations (i.e., a = 1,2,3…16; b = 1,2,3,….16) allows inversion, from which the desired error correlation matrix $\chi_{\mu\nu}$ may be deduced. Incorporating the necessary constraints so that the resultant matrix is positive and Hermitian, and that the process

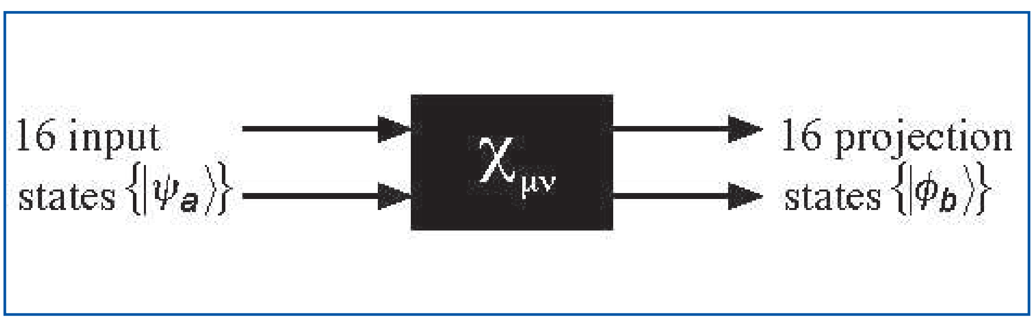

Figure 1— ***Essential idea for quantum process tomography.***

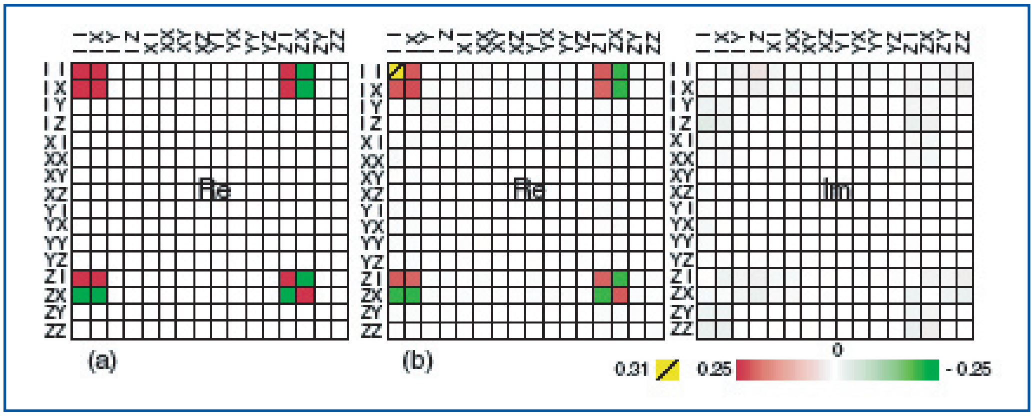

Figure 2—
Results of process tomography of a CNOT gate. The real elements of the error-correlation matrix for an ideal gate are shown in (a) the imaginary parts for the ideal gate are zero; (b) shows the real and imaginary parts of the measured process, deduced from data measured at the University of Queensland and processed using techniques developed at Los Alamos National Laboratory [1].

is trace-preserving turns out to require some rather complicated data analysis. The results are shown in Fig. 2.

This technique will be of considerable applicability for diagnosing devices as quantum information processing technologies evolve.

[1] J.L. O'Brien, G.J. Pryde, A. Gilchrist, D.F.V. James, N.K. Langford, T.C. Ralph, and A.G. White, "Quantum Process Tomography of a Controlled-NOT Gate," *Phys. Rev. Lett.* **93**, 080502 (2004).